THE IMPACT OF LOW FERTILITY RATE ON THE LEVEL OF PENSION

Uticaj niske stope fertiliteta na nivo penzije

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Abstract

This paper proposes an overlapping generations model along the lines of the papers by Fanti and Gori (2012). Its objective is to study the effect of low fertility on the level of pension respectively in the system PAYG and funded within the context of an exogenous fertility at long-run.

Keywords: Fertility rate, Pension system, Overlapping generations model.

1. Introduction

A number of studies have investigated the impact of population aging on the pension system. Accordingly, the declines in fertility rates and increased life expectancy have affected the solvency of pension system in many countries worldwide.

Theoretical models of overlapping generations literature have generally been interested in three effects of reduced fertility which are respectively the effect of intergenerational transfers, the effect of dilution of capital and the effect of the quality of the child. First, there currently studies such as Nishimura and Zhang (1992), Ehrlich and Lui (1998), Wigger (1999), Cremer et al. (2006) and Cigno (2010) showed that the


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decrease in fertility rates actually reduce the workforce in the future and therefore will create an imbalance between the number of contributors and pensioners. Second, Michel and Pestieau (1993) and Cigno (1993) have found that the decrease in the fertility rate causes respectively the increase in the capital per worker, output per worker and pension’s payout. Thirdly, two recent papers of Cremer et al. (2011) and Cipriani and Makris (2012) noted that the decline in fertility rates generates a better quality of education for children. This best quality education of children leads to increase future wages and tax revenue to the pension system.

But in our paper, we investigate the effect of reduced fertility on the level of pension in the case of a system of PAYG one hand, the other hand in the case of a funded pension. To achieve this study, we propose an overlapping generations model in the tradition of paper by Fanti and Gori (2012) and assume the rate of fertility is exogenously given.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyses how a low fertility affects the level of pension in the PAYG system at long-run. Section 4 studied this effect in the case of funded pension system. Section 5 concludes.

2. The model

Our model is inspired by the paper of Fanti and Gori (2012) in which agents live two periods.

Each individual born in\(^t\) consume successively \(c_t\) and \(c_{t+1}\). That’s why, the function of intertemporal utility of the representative agent is of log linear type and it is given by the following expression function:

\[
U_t = \ln c_t + \beta \ln c_{t+1}
\] (1)

During this first period, each worker pays a proportional tax on income from work, which is used to finance the pension system. Although the fertility rate is exogenous. Fertility is costly in terms of reduced labor income, as it takes time to raise her children. Thus, the budget constraint of the individual in the first period is written as follows:

\[
c_t + s_t + qw_t n = w_t (1 - \theta)
\] (2)

\(\theta\): The rate of pension paid by all workers.
\(qw_t n\): The cost of fertility, that is, the cost of raising children, which is a function of the fertility rate.
\(s_t\): Saving

In the second period, individuals are retired. During this period, retirees consume their savings from previous period increased interest accrued from \(t\) to \(t+1\) at the rate \(r_{t+1}\) on the one hand, on the other hand, the pension benefits \(P_{t+1}\).

\[
c_{t+1} = (1 + r_{t+1}) s_t + P_{t+1}
\] (3)

Maximization of utility subject to the budget constraints gives the following savings function:
This model supposes the existence of infinity of firms having the same production function and producing a homogeneous unique product. The production function is Cobb-Douglas: 

\[ Y_t = A K_t^\alpha L_t^{1-\alpha} \]

where \( A > 0 \) and \( \alpha \) is between 0 and 1. We assume that there is a total depreciation of capital at the end of each period and that the production is sold at unit price. This brings us to the following conditions of maximization profit:

\[ r_t = \alpha A k_t^{\alpha-1} - 1 \]  
\[ w_t = (1 - \alpha) A k_t^\alpha \]

With \( k_t = \frac{K_t}{L_t} \)

3. The impact of low fertility rate on pension level in the PAYG system

In this section, we consider a PAYG system. In a PAYG system, the contributions of the young are transferred to the old within the same period. Therefore, the pension spending at \( t \) is constrained by the total amount of contributions. In fact, the budget constraint of the government can then be written as:

\[ P_t = \theta w_t n \]  

Using the condition of government budget constraint in equation (4) allows us to obtain the following expression:

\[ s_t = \frac{\beta w_t (1 - \theta - qn)}{1 + \beta} - \frac{P_{t+1}}{(1 + \beta)(1 + r_{t+1})} \]  

At equilibrium in the market for goods and services, the saving is equal to the investment on the one hand; on the other hand, even the investment of period is equal to the capital stock of the next period. These both conditions that brings us to:

\[ n k_{t+1} = s_t \]

Inserting Equation (9) into Equation (8) gives:

\[ k_{t+1} = \frac{\beta w_t (1 - \theta - qn)}{n(1 + \beta)} - \frac{\theta}{1 + \beta} \frac{w_{t+1}}{1 + r_{t+1}} \]

Substituting Equations (5) and (6) in (10) by assuming that the expectations of individuals are perfect, this gives us the following dynamics of capital:
\[ k_{t+1} = \frac{\beta(1-\alpha)A(1-\theta-qn)}{n[\alpha(1+\beta) + \theta(1-\alpha)]} k_t^\alpha \]  

(11)

Steady-state occurs when \( k \) remains constant over time. We therefore find that the steady state value of \( k \) is given by:

\[ k^*(n) = \left\{ \frac{\beta\alpha(1-\alpha)A(1-\theta-qn)}{n[\alpha(1+\beta) + \theta(1-\alpha)]} \right\}^{\frac{1}{1-\alpha}} \]

(12)

From equation (12) we find that the decrease in fertility rate increases the capital stock. As the capital stock must be usually positive if \( n < (1-\theta)/q \).

After the determination of the steady state of the stock of capital, we interest in the determination of the level pension at long-run. For this, we combine equations (6), (7) and (12) to obtain:

\[ P_{PAYG} = \theta(1-\alpha)A \left\{ \frac{\beta\alpha(1-\alpha)A(1-\theta-qn)}{[\alpha(1+\beta) + \theta(1-\alpha)]} \right\}^{\frac{a}{1-\alpha}} \left( \frac{1-2\alpha}{n^{1-\alpha}} \right) \]

(13)

From equation (13) we visualize that the decline in fertility rate can cause two different effects on the level pension at long-run. The first effect is the decrease of the level pension because the number of contributors in the future will be increasingly reduced. The second effect is the increase of the level pension due to the rise of capital stock. But, according to the Equation (13) the second effect is most important that the first effect.

4. The impact of low fertility rate on pension level in the Funded system

In this section, we consider another type of pension system which is the funded pension system. This system consists for an individual to contribute today for his future retirement. During the period of retirement, each pensioner receives the mass of the contributions paid during the period of activity increased by their remuneration with interest rate. In fact, the budget constraint of the government can then be written as:

\[ P_t = (1+r_t)\theta \omega_{t-1} \]

(14)

Substituting equation (14) into equation (7) we obtain hence:

\[ s_t = \frac{w_t[\beta(1-\theta-qn)-\theta]}{(1+\beta)} \]

(15)

Within the framework of funded pension system, the equation of capital accumulation is expressed as follows \( nk_{t+1} = s_t + \theta \omega_t \). Thus, the exploitation of the
capital accumulation equation and two equations of wages and savings leads us to obtain the dynamics of the capital stock:

$$k_{t+1} = \frac{(1-\alpha)\beta(1-qn)}{n(1+\beta)} k_t^\alpha$$  \hspace{1cm} (16)

Thus, the steady-state of capital stock in the long term is of the following form:

$$k^*(n) = \left\{ \frac{(1-\alpha)\beta(1-qn)}{n(1+\beta)} \right\}^{\frac{1}{1-\alpha}}$$  \hspace{1cm} (17)

Subsequently, combining equations (6), (7) and (17) in order to determine the level of pension at long-run:

$$P_{funded} = \theta \alpha (1-\alpha)A^2 \left\{ \frac{(1-\alpha)\beta(1-qn)}{n(1+\beta)} \right\}^{\frac{2\alpha-1}{1-\alpha}}$$  \hspace{1cm} (18)

According to equation (18) we notice that the decline in fertility rates leads to decreased levels of pensions at long-run in a fully funded pension system. Therefore, the result found is contrary to the general findings that admit that the decline in fertility rate has a neutral effect on the level of pension in funded retirement system for example Cero (2007).

5. Conclusions

This paper presents a simple overlapping generation’s model with exogenous fertility rates and life expectancy constant in the line of paper Faint and Gori (2012). Thus, in addition the results of Faint and Gori (2012), we find that the decline in fertility rates leads to a reduction in the level of pension in the case of funded pension system contrary to what found a neutral effect as the Cero (2007).

References


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Apstrakt

Ovaj rad predlaže model preklapanja generacija na osnovu radova Fanti i Gori (2012). Njegov cilj je da se ispita uticaj niske stope fertiliteta na nivo penzije u sistemu PAYG i finansiranja u kontekstu egzogenog fertiliteta u dugom roku.

**Ključne reči:** stopa fertiliteta, pensioni sistem, model preklapanja generacija.